International Journal of Emerging Technologies in Engineering Research (IJETER)
Volume 2, Issue 3, October (2015) www.ijeter.everscience.org

Path Related Mean Square Cordial Graphs

Dr. A. Nellai Murugan
Department of Mathematics, V.O.Chidambaram College, Tuticorin, Tamilnadu, India.

S.Heerajohn
Department of Mathematics, V.O.Chidambaram College, Tuticorin, Tamilnadu, India.

Abstract – Let G = (V,E) be a graph with p vertices and q edges. A Mean Square Cordial Labeling of a Graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label $(\lceil (f(u))^2 + (f(u))^2 \rceil)/2$ where ⌈ x ⌉ (ceilex) is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. The graph that admits a Mean Square Cordial Labeling is called Mean Square Cordial Graph. In this paper, we proved that Path related graphs $Sp(P_n,K_{1,n})$, $(P_2 \cup mk_1)+N_2$ (m-odd), $P_n \Theta C_3$, $P_n@2k_{1,m}$, $P_n \otimes S_m$ (n-even) are Mean Square Cordial Graphs.

Index Terms – Mean Square Cordial Graph, Mean Square Cordial Labeling, 2000 Mathematics Subject classification 05C78.

1. INTRODUCTION

A graph G is a finite nonempty set of objects called vertices together with a set of unordered pairs of distinct vertices of G which is called edges. Each pair e = {u,v} of vertices in E is called edges or a line of G. In this paper, we proved that Path related graphs $Sp(P_n,K_{1,n})$, $(P_2 \cup mk_1)+N_2$ (m-odd), $P_n \Theta C_3$, $P_n@2k_{1,m}$, $P_n \otimes S_m$ (n-even) are mean square Cordial Graphs. For graph theory terminology, we follow [2].

2. PRELIMINARIES

Let G = (V,E) be a graph with p vertices and q edges. A Mean Square Cordial Labeling of a Graph G with vertex set V is a bijection from V to {0, 1} such that each edge uv is assigned the label $(\lceil (f(u))^2 + (f(u))^2 \rceil)/2$ where ⌈x⌉ (ceilex) is the least integer greater than or equal to x with the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1 and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1.

The graph that admits a Mean Square Cordial Labeling is called Mean Square Cordial Graph. In this paper, we proved that Path related graphs $Sp(P_n,K_{1,n})$, $(P_2 \cup mk_1)+N_2$ (m-odd), $P_n \Theta C_3$, $P_n@2k_{1,m}$, $P_n \otimes S_m$ (n-even) are Mean Square Cordial Graphs.

Definition: 2.1

$Sp(P_m, K_{1,n})$ is a graph in which the root of the star $K_{1,n}$ is attached at one end of the path P_m.

Definition: 2.2

The graph $(P_2 \cup mK_1) + N_2$ is a graph with vertex set $V=\{z_1,z_2,x_1,x_2\ldots x_m\} \cup \{y_1,y_2\}$ and edge set $\{[(y_1z_1),(y_1z_2),(y_2z_1),(y_2z_2),(z_1z_2)] \cup [(y_1x_i) \cup (y_2x_i) : 1 \le i \le m]\}$.

Definition: 2.3

The corona $G_1 \Theta G_2$ of two graphs G_1 and G_2 is defined as the graph G obtained by taking one copy of G_1 (which has P_1 points) and P_1 copies of G_2 and joining the i^{th} point of G_1 to every point in the i^{th} copy of G. A vertex of cycle C_3 attaching at every vertex of a path P_n is denoted by $P_n \Theta C_3$.

Definition: 2.4

$P_n@2k_{1,m}$ is a graph obtained from a path P_n by attaching root of a star $K_{1,m}$ at each pendent vertex of P_n.

Definition: 2.5

$P_n \otimes S_m$ is a graph obtaining from the path P_n by attaching root of a star S_m at every vertex of P_n

3. MAIN RESULTS

Theorem: 3.1

$Sp(P_n, K_{1,n})$ is Mean Square Cordial Graph.

Proof:

Let G be $Sp(P_n, K_{1,n})$

Let $V(G) = \{ u_i , v_i : 1 \le i \le n \}$

Let $E(G) = \{ [(u_iu_{i+1}) : 1 \le i \le n-1] \cup [(u_nv_i) : 1 \le i \le n] \}$

Define $f : V(G) \to \{0,1\}$

The vertex labeling are,

$f(u_i) = 0 , 1 \le i \le n$

$f(v_i) = 1 , 1 \le i \le n$

The induced edge labeling are,

$f^*(u_iu_{i+1}) = 0 , 1 \le i \le n-1$

$f^*(u_n v_i) = 1$, $1 \le i \le n$

Here, $v_f(1) = v_f(0)$ for all n and

$e_f(1) = e_f(0) + 1$ for all n

Therefore, The Graph G satisfies the conditions

$| v_f(1) - v_f(0) | \le 1$

$| e_f(1) - e_f(0) | \le 1$

Hence, $Sp(P_n, K_{1,n})$ is Mean Square Cordial Graph

For example, $Sp(P_3, K_{1,3})$ is Mean Square Cordial Graph as shown in figure 3.2.

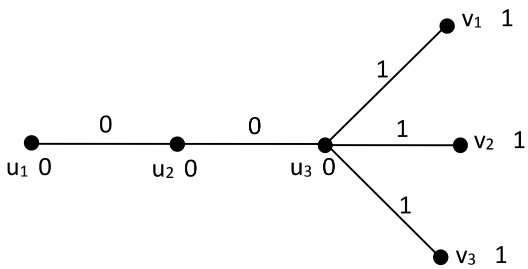


figure 3.2

Theorem: 3.3

$(P_2 \cup mk_1) + N_2$ (m-odd) is Mean Square Cordial Graph.

Proof:

Let G be $(P_2 \cup mk_1) + N_2$

Let $V(G) = \{ u , v, x, y , u_i : 1 \le i \le m \}$

Let $E(G) = \{ [(uv)] \cup [(ux)] \cup [(uy)] \cup [(vx)] \cup [(vy)] \cup [(xu_i) : 1 \le i \le m] \cup$

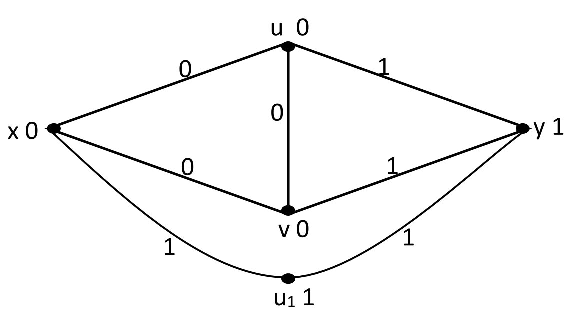
$[(yu_i) : 1 \le i \le m] \}$

Define $f : V(G) \to \{0,1\}$

Case: 1

When m = 1,

The labeling is,



Case: 2

When m > 1,

The vertex labeling are,

$f(u) = 0$

$f(v) = 0$

$f(x) = 0$

$f(y) = 0$

$$f(u_i) = \begin{cases} 1, & 1 \le i \le \frac{m+3}{2} \\ 0, & \frac{m+5}{2} \le i \le m \end{cases}$$

The induced edge labeling are,

$f^*(uv) = 0$

$f^*(ux) = 0$

$f^*(vx) = 0$

$f^*(uy) = 0$

$f^*(vy) = 0$

$$f^*(u_i x) = \begin{cases} 1, & 1 \le i \le \frac{m+3}{2} \\ 0, & \frac{m+5}{2} \le i \le m \end{cases}$$

$$f^*(u_i y) = \begin{cases} 1, & 1 \le i \le \frac{m+3}{2} \\ 0, & \frac{m+5}{2} \le i \le m \end{cases}$$

Here, $v_f(0) = v_f(1) + 1$ for all n and

$e_f(1) = e_f(0) + 1$ for all n

Therefore, The Graph G satisfies the conditions

$| v_f(1) - v_f(0) | \le 1$

$| e_f(1) - e_f(0) | \le 1$

Hence, The graph $(P_2 \cup mk_1) + N_2$ (m-odd) is Mean Square Cordial Graph

For example, $(P_2 \cup 5k_1) + N_2$ is Mean Square Cordial Graph as shown in figure 3.4.

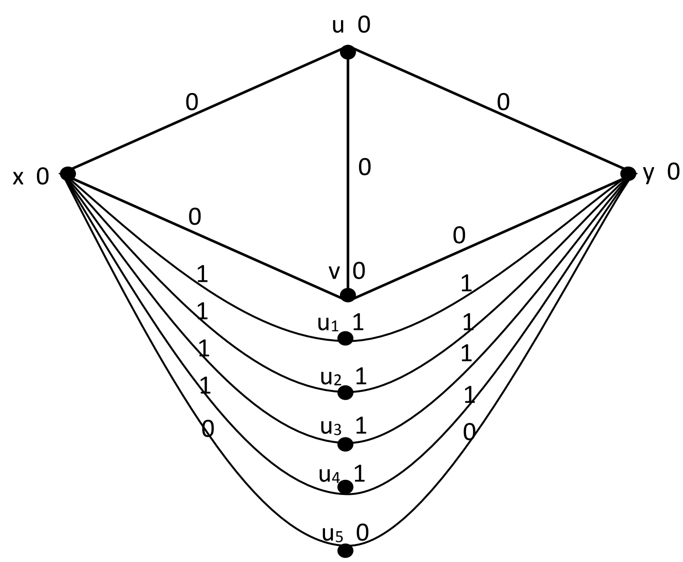


figure 3.4

Theorem: 3.5

$P_n \Theta C_3$ (n-even) is Mean Square Cordial Graph.

Proof:

Let G be $P_n \Theta C_3$

Let $V(G) = \{ u_i : 1 \leq i \leq n, u_{ij} : 1 \leq i \leq n, 1 \leq j \leq 2 \}$

Let $E(G) = \{ [(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i u_{ij}) : 1 \leq i \leq n, 1 \leq j \leq 2] \cup [(u_{i1} u_{i2}) : 1 \leq i \leq n] \}$

Define $f : V(G) \to \{0,1\}$

The vertex labeling are,

$$f(u_i) = \begin{cases} 0, 1 \leq i \leq \frac{n}{2} \\ 1, \frac{n+2}{2} \leq i \leq n \end{cases}$$

$$f(u_{ij}) = \begin{cases} 0, 1 \leq i \leq \frac{n}{2}, 1 \leq j \leq 2 \\ 1, \frac{n+2}{2} \leq i \leq n, 1 \leq j \leq 2 \end{cases}$$

The induced edge labeling are,

$$f^*(u_i u_{i+1}) = \begin{cases} 0, 1 \leq i \leq \frac{n-2}{2} \\ 1, \frac{n}{2} \leq i \leq n-1 \end{cases}$$

$$f^*(u_i u_{ij}) = \begin{cases} 0, 1 \leq i \leq \frac{n}{2}, 1 \leq j \leq 2 \\ 1, \frac{n+2}{2} \leq i \leq n, 1 \leq j \leq 2 \end{cases}$$

$$f^*(u_{i1} u_{i2}) = \begin{cases} 0, 1 \leq i \leq \frac{n}{2} \\ 1, \frac{n+2}{2} \leq i \leq n \end{cases}$$

Here, $v_f(1) = v_f(0)$ for all n and

$e_f(1) = e_f(0) + 1$ for all n

Therefore, The Graph G satisfies the conditions

$| v_f(1) - v_f(0) | \leq 1$

$| e_f(1) - e_f(0) | \leq 1$

Hence, $P_n \Theta C_3$ (n-even) is Mean Square Cordial Graph

For example, $P_4 \Theta C_3$ is Mean Square Cordial Graph as shown in figure 3.6.

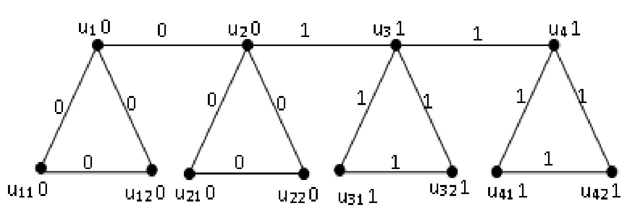


figure 3.6

Theorem: 3.7

$P_n \Theta C_3$ (n-odd) is Mean Square Cordial Graph.

Proof:

Let G be $P_n \Theta C_3$

Let $V(G) = \{ u_i : 1 \leq i \leq n, u_{ij} : 1 \leq i \leq n, 1 \leq j \leq 2 \}$

Let $E(G) = \{ [(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i u_{ij}) : 1 \leq i \leq n, 1 \leq j \leq 2] \cup [(u_{i1} u_{i2}) : 1 \leq i \leq n] \}$

Define $f : V(G) \to \{0,1\}$

The vertex labeling are,

$$f(u_i) = \begin{cases} 0, 1 \leq i \leq \frac{n+1}{2} \\ 1, \frac{n+3}{2} \leq i \leq n \end{cases}$$

$$f(u_{i1}) = \begin{cases} 0, 1 \leq i \leq \frac{n+1}{2} \\ 1, \frac{n+3}{2} \leq i \leq n \end{cases}$$

$$f(u_{i2}) = \begin{cases} 0, 1 \leq i \leq \frac{n-1}{2} \\ 1, \frac{n+1}{2} \leq i \leq n \end{cases}$$

The induced edge labeling are,

$$f^*(u_i u_{i+1}) = \begin{cases} 0, 1 \leq i \leq \frac{n-1}{2} \\ 1, \frac{n+1}{2} \leq i \leq n-1 \end{cases}$$

$$f^*(u_i u_{i1}) = \begin{cases} 0, 1 \leq i \leq \frac{n+1}{2} \\ 1, \frac{n+3}{2} \leq i \leq n \end{cases}$$

$$f^*(u_i u_{i2}) = \begin{cases} 0, 1 \leq i \leq \frac{n-1}{2} \\ 1, \frac{n+1}{2} \leq i \leq n \end{cases}$$

$$f^*(u_{i1} u_{i2}) = \begin{cases} 0, 1 \leq i \leq \frac{n-1}{2} \\ 1, \frac{n+1}{2} \leq i \leq n \end{cases}$$

Here, $v_f(1) + 1 = v_f(0)$ for all n and

$e_f(0) + 1 = e_f(1)$ for all n.

Therefore, The Graph G satisfies the conditions

$| v_f(1) - v_f(0) | \leq 1$

$| e_f(1) - e_f(0) | \leq 1$

Hence, $P_n \Theta C_3$ (n-odd) is Mean Square Cordial Graph

For example, $P_3 \Theta C_3$ is Mean Square Cordial Graph as shown in figure 3.8.

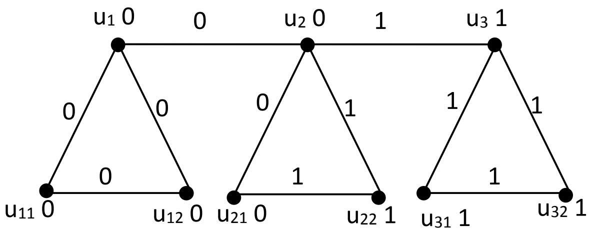


figure 3.8

Theorem: 3.9

$P_n@2k_{1,m}$ (n-even) is Mean Square Cordial Graph.

Proof:

Let G be $P_n@2k_{1,m}$

Let V(G) = { (u_i , w_i) : $1\leq i\leq m$, v_i : $1\leq i\leq n$ }

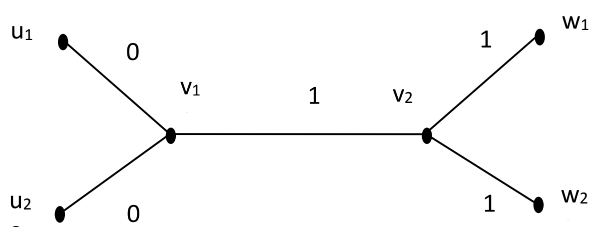
Let E(G) = { [(u_iv_1) : $1\leq i\leq m$] ∪ [(v_iv_{i+1}) : $1\leq i\leq n-1$] ∪ [(v_nw_i) : $1\leq i\leq m$] }

Define f : V(G)→ {0,1}

Case: 1

when n = 2,

The labeling is,



Case: 2

when n > 2 and n ≡ 0 (mod 2),

The vertex labeling are,

$f(u_i) = 0$, $1\leq i\leq m$

$f(w_i) = 1$, $1\leq i\leq m$

$$f(v_i) = \begin{cases} 0\,, 1 \le i \le \frac{n}{2} \\ 1\,,\ \frac{n+2}{2} \le i \le n \end{cases}$$

The induced edge labeling are,

$f^*(u_iv_1) = 0$, $1\leq i\leq m$

$f^*(v_nw_i) = 1$, $1\leq i\leq m$

$$f^*(v_iv_{i+1}) = \begin{cases} 0\,, 1 \le i \le \frac{n-2}{2} \\ 1\,,\ \frac{n}{2} \le i \le n-1 \end{cases}$$

Here, $v_f(1) = v_f(0)$ for all n and

$e_f(1) = e_f(0) + 1$ for all n

Therefore, The Graph G satisfies the conditions

$| v_f(1) – v_f(0) | \leq 1$

$| e_f(1) – e_f(0) | \leq 1$

Hence, $P_n@2k_{1,m}$ (n-even) is Mean Square Cordial Graph

For example, $P_4@2k_{1,3}$ is Mean Square Cordial Graph as shown in figure 3.10.

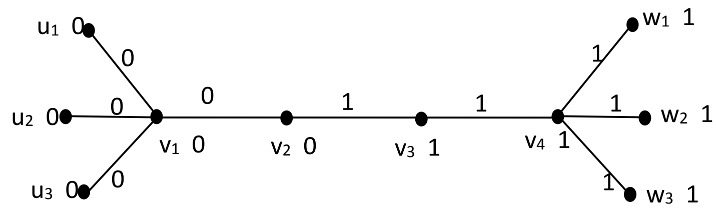


figure 3.10

Theorem: 3.11

$P_n@2k_{1,m}$ (n-odd) is Mean Square Cordial Graph.

Proof:

Let G be $P_n@2k_{1,m}$

Let V(G) = { (u_i , w_i) : $1\leq i\leq m$, v_i : $1\leq i\leq n$ }

Let E(G) = { [(u_iv_1) : $1\leq i\leq m$] ∪ [(v_iv_{i+1}) : $1\leq i\leq n-1$] ∪ [(v_nw_i) : $1\leq i\leq m$] }

Define f : V(G)→ {0,1}

The vertex labeling are,

$f(u_i) = 0$, $1\leq i\leq m$

$f(w_i) = 1$, $1\leq i\leq m$

$$f(v_i) = \begin{cases} 0\,, 1 \le i \le \frac{n+1}{2} \\ 1\,,\ \frac{n+3}{2} \le i \le n \end{cases}$$

The induced edge labeling are,

$f^*(u_iv_1) = 0$, $1\leq i\leq m$

$f^*(v_nw_i) = 1$, $1\leq i\leq m$

$$f^*(v_iv_{i+1}) = \begin{cases} 0\,, 1 \le i \le \frac{n-1}{2} \\ 1\,,\ \frac{n+1}{2} \le i \le n-1 \end{cases}$$

Here, $v_f(0) = v_f(1) + 1$ for all n and

$e_f(0) = e_f(1)$ for all n

Therefore, The Graph G satisfies the conditions

$| v_f(1) - v_f(0) | \leq 1$

$| e_f(1) - e_f(0) | \leq 1$

Hence, $P_n@2k_{1,m}$ (n-odd) is Mean Square Cordial Graph

For example, $P_3@2k_{1,2}$ is Mean Square Cordial Graph as shown in figure 3.12.

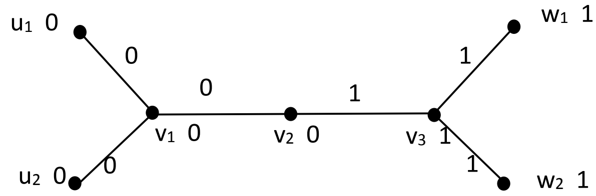


figure 3.12

Theorem: 3.13

$P_n \otimes S_m$ (n-even) is Mean Square Cordial Graph.

Proof:

Let G be $P_n \otimes S_m$

Let $V(G) = \{ u_i , u_{ij} : 1 \leq i \leq n , 1 \leq j \leq m \}$

Let $E(G) = \{ [(u_i u_{i+1}) : 1 \leq i \leq n-1] \cup [(u_i u_{ij}) : 1 \leq i \leq n , 1 \leq j \leq m] \}$

Define $f : V(G) \rightarrow \{0,1\}$

The vertex labeling are,

$$f(u_i) = \begin{cases} 0 , 1 \leq i \leq \frac{n}{2} \\ 1 , \frac{n+2}{2} \leq i \leq n \end{cases}$$

$$f(u_{ij}) = \begin{cases} 0 , 1 \leq i \leq \frac{n}{2} \ ,1 \leq j \leq m \\ 1 , \frac{n+2}{2} \leq i \leq n ,1 \leq j \leq m \end{cases}$$

The induced edge labeling are,

$$f^*(u_i u_{i+1}) = \begin{cases} 0 , 1 \leq i \leq \frac{n-2}{2} \\ 1 , \frac{n}{2} \leq i \leq n-1 \end{cases}$$

$$f^*(u_i u_{ij}) = \begin{cases} 0 , 1 \leq i \leq \frac{n}{2} , 1 \leq j \leq m \\ 1 , \frac{n+2}{2} \leq i \leq n ,1 \leq j \leq m \end{cases}$$

Here, $v_f(0) = v_f(1)$ for all n and

$e_f(1) = e_f(0) + 1$ for all n

Therefore, The Graph G satisfies the conditions

$| v_f(1) - v_f(0) | \leq 1$

$| e_f(1) - e_f(0) | \leq 1$

Hence, $P_n \otimes S_m$(n – even) is mean square cordial graph

For example, $P_4 \otimes S_3$ is mean square cordial graph as shown in figure 3.14.

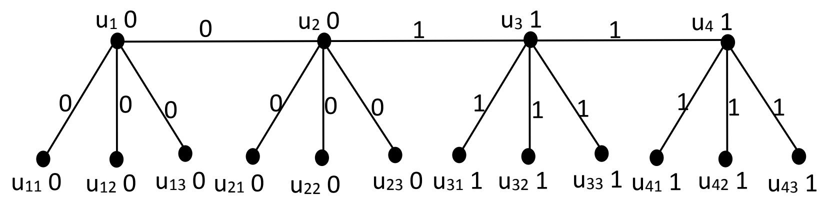


figure 3.14

4. CONCLUSION

Cordial graphs based on digital principles. Mean graphs has its own advantages. Combining mean square and cordial may yield better applications. Here it is identified some graphs are mean square cordial.

REFERENCES

[1] Gallian. J.A,A *Dynamic Survey of Graph Labeling*, The Electronic Journal of Combinotorics 6(2001)#DS6.

[2] Harary,F.(1969), *Graph Theory*, Addision – Wesley Publishing Company Inc, USA.

[3] A.Nellai Murugan *(*September 2011), *Studies in Graph theory- Some Labeling Problems in Graphs and Related topics,* Ph.D Thesis.

[4] A.Nellai Murugan and V.Baby Suganya, Cordial labeling of path related splitted graphs, Indian Journal of Applied Research ISSN 2249 – 555X,Vol.4, Issue 3, Mar. 2014, ISSN 2249 – 555X , PP 1-8. I .F . 2.1652

[5] A.Nellai Murugan and M. Taj Nisha, A study on divisor cordial labelling of star attached paths and cycles, Indian Journal of Research ISSN 2250 –1991,Vol.3, Issue 3, Mar. 2014, PP 12-17. I .F . 1.6714.

[6] A.Nellai Murugan and V.Brinda Devi, A study on path related divisor cordial graphs International Journal of Scientific Research, ISSN 2277–8179,Vol.3, Issue 4, April. 2014, PP 286 - 291. I .F . 1.8651.

[7] A.Nellai Murugan and A Meenakshi Sundari, On Cordial Graphs International Journal of Scientific Research, ISSN 2277–8179,Vol.3, Issue 7 ,July. 2014, PP 54-55. I .F . 1.8651

[8] A.Nellai Murugan and A Meenakshi Sundari, Results on Cycle related product cordial graphs, International Journal of Innovative Science, Engineering & Technology , ISSN 2348-7968,Vol.I, Issue 5 ,July. 2014, PP 462-467.IF 0.611

[9] A.Nellai Murugan and P.Iyadurai Selvaraj, Cycle and Armed Cup cordial graphs, International Journal of Innovative Science, Engineering & Technology , ISSN 2348- 7968,Vol.I, Issue 5 ,July. 2014,PP 478-485. IF 0.611

[10] A.Nellai Murugan and G.Esther, Some Results on Mean Cordial Labelling , International Journal of Mathematics Trends and Technology ,ISSN 2231-5373,Volume 11, Number 2,July 2014,PP 97-101.

[11] A.Nellai Murugan and A Meenakshi Sundari, Path related product cordial graphs, International Journal of Innovation in Science and Mathematics , ISSN 2347-9051,Vol 2., Issue 4 ,July 2014, PP 381-383

[12] A.Nellai Murugan and P. Iyadurai Selvaraj, P*ath Related Cup Cordial graphs*, Indian Journal of Applied Research, ISSN 2249 –555X,Vol.4, Issue 8, August. 2014, PP 433-436.

[13] A.Nellai Murugan, G.Devakiriba and S. Navaneetha Krishnan, Star Attached Divisor cordial graphs, International Journal of Innovative Science, Engineering & Technology, ISSN 2348-7968,Vol.I, Issue 6 ,August. 2014, PP 165-171.

[14] A.Nellai Murugan and G. Devakiriba, Cycle Related Divisor Cordial Graphs, International Journal of Mathematics Trends and Technology , ISSN 2231-5373,Volume 12, Number 1,August 2014,PP 34-43.
[15] A.Nellai Murugan and V.Baby Suganya, A study on cordial labeling of Splitting Graphs of star Attached C_3 and $(2k+1)C_3$ ISSN 2321 8835, Outreach , A Multi Disciplinary Refreed Journal, Volume . VII, 2014, 142 -147. I.F 6.531
[16] A.Nellai Murugan and V.Brinda Devi , A study on Star Related Divisor cordial Graphs ,ISSN 2321 8835, Outreach , A Multi Disciplinary Refreed Journal, Volume . VII, 2014, 169-172. I.F 6.531
[17] A.Nellai Murugan and M. Taj Nisha, A study on Divisor Cordial Labeling Star Attached Path Related Graphs, ISSN 2321 8835, Outreach , A Multi Disciplinary Refreed Journal, Volume . VII, 2014, 173-178. I.F 6.531 .
[18] .A .Nellai Murugan and V .Sripratha, Mean Square Cordial Labelling , International Journal of Innovative Research & Studies, ISSN 2319-9725 ,Volume 3, Issue 10Number 2 , October 2014, PP 262-277.
[19] A.Nellai Murugan and G. Esther , Path Related Mean Cordial Graphs , Journal of Global Research in Mathematical Archive , ISSN 2320 5822 , Volume 02, Number 3,March 2014,PP 74-86.
[20] A. Nellai Murugan and A. Meenakshi Sundari , Some Special Product Cordial Graphs, Proceeding of the UGC Sponsored National Conference on Advances in Fuzzy Algebra , Fuzzy Topology and Fuzzay Graphs , Journal ENRICH , ISSN 2319-6394, January 2015, PP 129-141.
[21] L. Pandiselvi ,S.Navaneethakrishan and A. Nellai Murugan ,Fibonacci divisor Cordial Cycle Related Graphs, Proceeding of the UGC Sponsored National Conference on Advances in Fuzzy Algebra , Fuzzy Topology and Fuzzay Graphs , Journal ENRICH , ISSN 2319-6394, January 2015, PP 142-150.

Authors

Dr. A. Nellai Murugan, Associate Professor, S.S.Pillai Centre for Research in Mathematics, Department of Mathematics, V.O.Chidambaram College, Thoothukudi. College is affiliated to Manonmanium sundaranar University, Tirunelveli-12, TamilNadu. He has thirty two years of Post Graduate teaching experience in which twelve years of Research experience. He is guiding six Ph.D Scholars. He has published more than seventy research papers in reputed national and international journals.

S.Heerajohn, She is a full time M.Sc Student, Department of Mathematics, V.O. Chidambaram College, Tuticorin. Her Project in the second year is labeling in Graph. She published two Research Article and Two more in communication.